Econ Theory (2016) 61:215–239
DOI 10.1007/s00199-015-0916-5

CrossMark

SYMPOSIUM

Credit search and credit cycles

Feng Dong[1] · **Pengfei Wang**[2] · **Yi Wen**[3,4]

Received: 10 December 2014 / Accepted: 13 September 2015 / Published online: 26 September 2015

Abstract The supply and demand of credit are not always well aligned, as is reflected in the countercyclical excess reserve-to-deposit ratio and interest spread between the lending rate and the deposit rate. We develop a search-based theory of credit allocations to explain the cyclical fluctuations in both bank reserves and interest spread. We show that search frictions in the credit market can naturally explain the countercyclical bank reserves and interest spread, as well as generate endogenous business cycles driven primarily by the cyclical utilization rate of credit resources, as long conjectured by the Austrian school of the business cycle. In particular, we show that credit search can lead to endogenous local increasing returns to scale and variable capital utilization in a model with constant returns to scale production technology and matching functions, thus providing a microfoundation for the indeterminacy literature of Benhabib and Farmer (J Econ Theory 63(1):19–41, 1994) and Wen (J Econ Theory 81(1):7–36, 1998).

✉ Feng Dong
fengdong@sjtu.edu.cn

Pengfei Wang
fwang@ust.hk

Yi Wen
yi.wen@stls.frb.org

1 Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai, China

2 Department of Economics, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

3 Research Department, Federal Reserve Bank of St. Louis, Saint Louis, MO, USA

4 School of Economics and Management, Tsinghua University, Beijing, China

Keywords Search frictions · Credit utilization · Credit rationing · Self-fulfilling prophecy · Business cycles

JEL Classification E22 · E32 · G21

1 Introduction

The role of financial intermediation and credit supply in driving and amplifying the business cycle has long been analyzed in the history of economic thought at least since the Austrian school. The Austrian theory of the business cycle emphasizes bank issuance of credit as the main cause of economic fluctuations. It asserts that the banking sector's excessive credit supply and low interest policy (with the rate of loanable funds below the natural rate) drive firms' investment boom, and its tight credit and interest rate policy (with the rate of loanable funds above the natural rate) generate economic slump.

The history of financial crisis appears consistent with the Austrian theory. A notable feature of the financial crisis in 2007, for example, is that before the financial crisis both the bank interest rate and the reserve-to-deposit ratio were excessively low in the economic boom period leading up to the financial crisis. On the verge of the financial crisis in late 2007 and early 2008, however, demand for credit on the firm side grew excessive (as reflected by the interest rate hike) and credit supply on the bank side tightened (as evidenced by the significantly increased excess reserve-to-deposit ratio). For example, according to a survey by Campello et al. (2010), about 60 % of US chief financial officers believe that their firms are financially constrained. Among them, 86 % claim that they pass up attractive investment opportunities due to the inability to raise external financing. Yet, banks were building up their cash positions at unprecedented speed. Bank excessive reserves skyrocketed from 1.93 to 1043.30 billion from the second quarter of 2008 to the second quarter of 2010, while the growth of bank loans plunged 37.17 % during the same period. Corresponding to the tight credit supply was the high interest rate on bank loans. Thus, we observe in Fig. 1 a countercyclical movement in the excess reserve-to-deposit ratio (dashed line, re-scaled) and in Fig. 2 a countercyclical movement in interest rate spread between the loan rate and the three-month treasury bill rate (dashed line).

However, correlation is not causation. It is unclear from the figures whether the observed credit cycle and interest movements are endogenous outcomes (symptoms) of the business cycle or the cause of it. This paper tries to shed light on these critical issues.

Specifically, we provide a search-based financial intermediation theory to explain the observed countercyclical excess reserve-to-deposit ratio and countercyclical interest rate spread in the data. We begin by noticing that in the real world, there are always agents with savings and agents with investment projects, but the demand side of the credit market (e.g., firms) and the supply side of the credit market (e.g., households and banks) must overcome search frictions to channel funds from savers to firms. This is especially the case in developing countries where financial markets are highly underdeveloped such that underground credit search market and shadow banking are

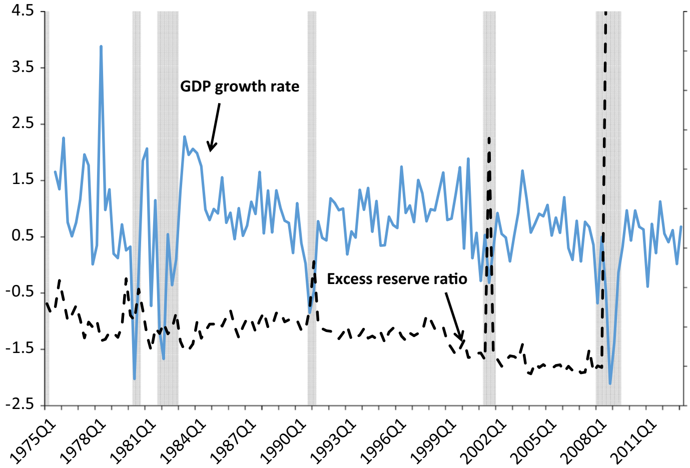

Fig. 1 GDP growth rate and excess reserve ratio. Data source: Federal Reserve Economic Data (FRED)

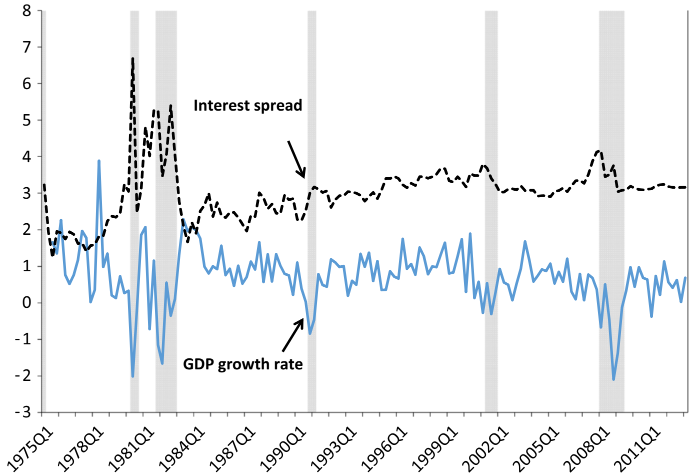

Fig. 2 GDP growth rate and interest spread (loan rate minus three-month treasury bill rate). Data source: FRED

pervasive. We show that such search frictions can indeed lead to a countercyclical excess reserve-to-deposit ratio and countercyclical interest rate spread. More importantly, they can also lead to endogenous business cycles driven by self-fulfilling beliefs about the tightness of credit conditions in the credit market. Such coordinated beliefs produce economic fluctuations through affecting the effective utilization rate of the aggregate credit resources. Moreover, our calibration analysis reveals that an endogenous multiplier-accelerator propagation mechanism that is rooted in credit search is not only theoretically appealing but also empirically plausible. The model captures many of the insights and predictions from the Austrian theory.

Our model also sheds light on the issue of credit rationing. Credit rationing is not only of theoretic interest, but also plays a non-trivial role in real-world firm financing.

As documented in Becchetti et al. (2009), approximately 20.24 % of firms are subject to credit rationing in Italy. However, the literature on credit rationing is extremely thin despite the seminal work of Stiglitz and Weiss (1981). Therefore, in addition to shedding light on the cyclical behavior of bank reserves and interest spread and credit-led business cycles, our search-theoretic approach also provides a shortcut to quantitatively study the business cycle property of credit rationing.

To highlight the relevance of credit market search frictions to the business cycle, our framework is by design kept extremely simple with off-the-shelf search and matching frictions, yet the results can be very powerful. Specifically, the benchmark model features three types of agents: a representative household with a continuum of *ex ante* identical members (depositors), a representative financial intermediary (bank) with a continuum of *ex ante* identical loan officers, and a continuum of firms. The banking sector accepts deposits from the household and then lends credit to firms through search and matching. We assume search frictions exist both between the household and the banking sector and between the banking sector and firms. We will show which search frictions are more critical to generating self-fulfilling credit cycles.

Similar to the standard Diamond–Mortensen–Pissarides (DMP) search and matching model of unemployment, our model features aggregate matching functions that determine the number of credit relationships between depositors and financial intermediaries, and between loan officers and firms. Such search frictions create unutilized credit resources in equilibrium, analogous to the unemployed labor force in the DMP model. For example, when bank deposits are not matched with firms, they become idle (excess reserves) in the banking system, while firms that are not matched with loans are considered as being denied for credit. This simple matching framework then provides a quantitative framework to analyze and explains the coexistence of excess reserves and credit rationing in the data. Since a booming economy encourages more costly search in the credit market, it increases the probability of matching credit resources. As a result, the reserve-to-deposit ratio is countercyclical over the business cycle. In addition, since the deposit rate facing the household sector is determined mainly by time preference (the natural rate) and the lending rate facing firms mainly by credit availability and firms' credit demand, the spread between the loan rate and the deposit rate may also be countercyclical under aggregate shocks. Thus, our framework provides a natural interpretation of the concepts of natural rate and loanable rate of interest introduced by the Austrian school.

In addition, we show that an elastic supply of credit due to a variable utilization rate of existing credit resources under search and matching can lead to *local* increasing returns to scale (IRS) in the aggregate production function even though the underlying production and matching technologies both exhibit constant returns to scale (CRS). This endogenous source of local IRS arising from procyclical credit utilization can in turn generate local indeterminacy and self-fulfilling credit cycles that feature a powerful multiplier-accelerator propagation mechanism.

In our model, an anticipated increase in credit supply from the banking sector (in the absence of any fundamental shocks) would entice firms to increase search efforts, resulting in more credit matches. Therefore, more capital would be channeled from the financial sector to firms, provided that the cost of borrowing does not increase so much as to discourage entry. With more loans (capital) in hand, firms can increase production by hiring more labor, so households' labor supply would also increase, leading to higher aggregate income and household savings. If the increase in household savings is large enough, it would then stimulate bank deposits and boost banks' credit supply without creating too much upward pressure on the loanable funds rate, validating firms' initial optimistic expectations about credit conditions. But the process does not stop here. Because of higher deposits, competition among banks will reduce the loan rate, which will attract more firms to the credit market in search of loans. A higher rate of firm entry in turn will further increase the matching probability, raising the effective capital stock used in firms' production even more. Consequently, the economy will enter a persistent boom period (after an initial shock) that features all the symptoms of a credit cycle described by the Austrian school. According to the Austrian school, a seemingly easy credit policy is associated with a higher lending volume, higher aggregate production, and higher employment, which in turn generates higher household consumption, savings, and bank deposits with possibly further lowered interest rate. However, in the absence of true productivity (technological) growth, such an economic boom is not sustainable in the long run, because the limited credit resources in the economy will eventually be exhausted under concave production technologies. The loanable funds rate will eventually rise to a high enough level to clear the credit market and end the boom. Once the boom ends, a prolonged recession will follow as the above multiplier-accelerator feedback mechanism reverses itself.

Hence, our model produces genuine credit cycles described by the Austrian economists: An economic boom led by credit expansion will plant the seed for an economic downturn, and a downturn will plant the seed for the next boom.

Technically speaking, the persistence of an endogenous credit cycle lies in the local IRS, which originates from a subtle pecuniary externality (based on search and firm entry) instead of the technological IRS based on production externality (as in Benhabib and Farmer 1994). Under local IRS, a proportionate increase in household labor supply and savings would cause firms' effective capital stock and aggregate production to increase more than proportionally. Also, as the probability of matching credit increases, the banking sector is able to pay a proportionately higher deposit rate relative to the loan rate to attract household deposits, leading to countercyclical interest spread. This will increase the rate of return to household savings even for those households that do not increase their saving rate, and decrease the cost of credit (interest payments) even for those firms that do not increase their borrowings, further reinforcing the positive feedback loop among saving, credit, and investment, as emphasized by the Austrian school. The IRS is local in nature because the utilization rate of credit resources in the aggregate economy cannot exceed 100 %. Once the utilization rate reaches 100 %, the highly elastic supply of credit would cease to exist and the model economy would become identical to that in a standard real business cycle (RBC) model.

The endogenous local IRS in our model are appealing for several reasons. First, it is consistent with CRS production technologies. Second, aggregate demand shocks (such as preference shocks or government spending shocks) are now able to generate positive business cycle comovement among aggregate consumption, investment, and output. Demand shocks are widely believed to be important sources of business cycles, yet in standard RBC models they generally produce a negative comovement between consumption and investment. Third, the standard RBC model has been criticized for requiring large technology shocks to produce realistic business cycles (see King and Rebelo (1999) for a survey of the literature). Thanks to the endogenous IRS in our model, small fundamental shocks (either demand or supply shocks, including news shocks) can generate large business cycle fluctuations with positive comovements, without assuming various types of adjustments costs and special utility functions. Fourth, our model can generate indeterminacy and self-fulfilling business cycles with hump-shaped output responses without productive externalities as in the model of Benhabib and Farmer (1994) and the variable capacity utilization model of Wen (1998).

Our paper is related to several strands of literature. First, search friction is in line with approaches proposed by Den Haan et al. (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013). These researchers have explored the implication of credit search on the macroeconomy, but have not studied the possibility of indeterminacy. For simplicity and tractability, they have assumed linear utility. In contrast, we incorporate credit search friction into an otherwise standard RBC model. This allows us to study a richer set of economic variables of interest. Our paper is also inspired by search frictions in goods market such as Bai et al. (2012), and by search-theoretic models of asset trading such as Duffie et al. (2005) and Lagos and Rocheteau (2009). Recently, Cui and Radde (2014) incorporate this line of research into a dynamic general equilibrium model and show that it can explain the interesting flight-to-liquidity phenomenon observed in the Great Recession.

Our model also provides a microfoundation for the Benhabib–Farmer (1994) model with IRS and the Wen (1998) model with a variable rate of capital utilization under IRS. We show that search frictions in the credit market can generate an economic structure isomorphic to the Benhabib–Farmer–Wen model with both increasing returns and elastic capacity utilization, yet without assuming IRS in the production technology of firms. Our paper is also closely related to several recent works on business cycles that are self-fulfilling due to credit market frictions, such as Gertler and Kiyotaki (2013), Miao and Wang (2012), Azariadis et al. (2014), Benhabib and Wang (2013), Benhabib et al. (2014a, b), Pintus and Wen (2013), and Liu and Wang (2014).

Finally, our model is similar in spirit to Acemoglu (1996), who shows that search friction in labor markets generates increasing returns to human capital accumulation in a two-period model. In his model, the workers must make human capital investments before they can enter the labor market. An increase in the average human capital investment induces more physical investments from firms. So even those of workers who have not increased their human capital will earn a higher return on their human capital if matched with firms. In other words, search friction produces a positive pecuniary externality similar to the mechanism in our model. However, unlike Acemoglu (1996), we focus on search in credit markets and explore its implication on indeterminacy and self-fulfilling expectation-driven business cycles in an infinite-period model.

The rest of the paper is organized as follows. Section 2 and Sect. 3 lay out the baseline model and examine its key properties, respectively. Section 4 studies the model's business cycle implications under calibrated parameter values. Section 5 extends the baseline model, and Sect. 6 concludes the paper. The omitted proofs appear in the "Appendix."

2 Model

2.1 Environment

Time is continuous. The economy is populated by three types of agents: a representative household composed of a continuum of depositors, a representative bank (financial intermediary) composed of a continuum of clerks or loan officers, and a continuum of firms. We assume perfect competition in all sectors. The household owns capital and firms, makes decisions on labor supply and consumption, and deposits savings into the banking system, which channels capital (in the form of loans) to firms. To break the conventionally assumed accounting identity between aggregate household saving and firm investment (which was criticized by the Austrian school and Keynes), we assume search frictions among the three types of agents so that in equilibrium aggregate household saving does not automatically equal firm investment, but instead imposes an upper limit on firm investment for any given interest rate.

The timeline of events in an interval from t to $t + \mathrm{d}t$ is as follows. First, loan officers search for depositors to collect bank reserves, or alternatively, depositors search for loan officers to deposit their savings (carried over from the last period) into the banking system. So there is search and matching friction between depositors and the banking system. Without loss of generality, we assume that the household pays for the search costs. After collecting deposits, the loan officers and firms engage in random search and matching.[1] Again we assume that firms pay for the search costs. In order to enter the credit market, however, a firm needs to pay a fixed cost. If a firm is matched successfully and obtains a loan, the trading surplus is split between the firm and the bank and the credit relationship dissolves by the end of the period.[2] After obtaining a loan (capital), firms can start producing goods by hiring labor in the spot market. The number of active firms engaging in production is then determined by the free entry condition: The expected surplus from a successful match equals the fixed search cost of entering the credit market. Finally, the household pools wage and profit incomes from the bank and firms and then makes a decision on consumption and capital accumulation (next-period savings). The whole process is repeated in the next time interval between $t + \mathrm{d}t$ and $t + 2\mathrm{d}t$.

To facilitate the analysis, we assume that all depositors from the household are *ex ante* identical and assigned with the same amount of credit resources to be randomly matched with a continuum of *ex ante* identical loan officers. Any unmatched savings are kept as inventories and carried over to the next period by the household. Analogously, after collecting deposits from the household, all loan officers are assigned with the same amount of credit resources available to be randomly matched with firms. Any unmatched loans are counted as excess reserves and transferred as a lump sum back to the households at the end of each period.[3]

[1] See Sect. 5.3 for an alternative setup with competitive search. All the qualitative results derived in this paper are preserved under competitive search.

[2] We address the issue of long-term credit relationships in a companion paper, but the basic results hold.

[3] In a follow-up project, we study the case with required reserves and interbank lending.

Specifically, denote the total savings of the household by S. Due to search frictions, only $\widetilde{S} < S$ units of savings are successfully matched and deposited into the banking system. After that, each loan officer is assigned with an equal fraction of the $\widetilde{S}$ units of deposits and goes out searching for potential borrowers (firms).

We show that such a simple setup leads to a simple dynamic system that can generate (i) a countercyclical excess reserve-to-deposit ratio, (ii) countercyclical interest rate spread between the loan rate and the deposit rate, and (iii) self-fulfilling business cycles with strong amplification and propagation mechanisms.

2.2 Deposit search

We first consider search frictions between the household and the banking system. The matching function between household members and bank clerks is $M^H(x_t H_t, B_t) = \gamma_H (x_t H_t)^{\varepsilon_H} B_t^{1-\varepsilon_H}$, where x_t is the search effort chosen by household depositors. There is a unit measure of household depositors and bank clerks, i.e., $H_t = B_t = 1$.[4] Thus, the matching probability is given by

$$e_t = \frac{M^H(x_t H_t, B_t)}{H_t} = \gamma_H x_t^{\varepsilon_H}. \tag{1}$$

Denoting the time interval by $\mathrm{d}t$, the budget constraint of the household can be written as

$$C_t \mathrm{d}t + S_{t+\mathrm{d}t} = \left[e_t\left(1 + R_t^d \mathrm{d}t\right) - \phi^H x_t \mathrm{d}t\right] S_t + (1 - e_t) S_t + W_t N_t \mathrm{d}t + \Pi_t \mathrm{d}t \tag{2}$$

subject to equation (1), where C_t denotes consumption, S_t total savings, e_t the fraction of aggregate savings successfully deposited into the banking system, R_t^d the deposit rate promised by the bank, x_t the search effort made by the household, $W_t N_t$ the wage income and Π_t the profit income from banks and firms, which are to be specified later. Note that the first term on the right-hand side (RHS) pertains to the cross-rate of return to deposits per unit of savings, $e_t\left(1 + R_t^d\right)$, after subtracting the search cost per unit of savings in hand, $\phi^H x_t$. That is, we assume that the search cost for each depositor is proportional to his/her effort x_t and the stock of savings in hand.[5] The second term on the RHS, $(1 - e_t) S_t$, is the total idle (unmatched) credit resources, which is also the unmatched stock of savings kept by the household.

The first-order condition (FOC) of the effort choice is given by $x_t = [(\frac{\gamma_H \varepsilon_H}{\phi_H}) R_t^d]^{\frac{1}{1-\varepsilon_H}}$. In turn, the aggregate utilization rate of household savings is $e_t = \gamma_H [(\frac{\gamma_H \varepsilon_H}{\phi_H}) R_t^d]^{\frac{\varepsilon_H}{1-\varepsilon_H}}$. Because of the search costs, we can derive a pseudo-"depreciation" function of the

[4] Our results would be strengthened if we allow H_t and B_t to vary by costly entry as an additional margin of adjustment.

[5] We choose this proportional cost function for comparison with the fixed search cost on the firm side (to be specified below). This way we can show which form of search costs leads to local IRS in our model.

stock of savings as follows. Denoting $\delta^0 \equiv \frac{\phi_H(1+\kappa)}{\gamma_H^{1+\kappa}}$ and $\kappa \equiv \frac{1}{\varepsilon_H} - 1$, we can define

$$\delta(e_t) \equiv \phi^H \left(\frac{e_t}{\gamma_H}\right)^{\frac{1}{\varepsilon_H}} \equiv \delta^0 \left(\frac{e_t^{1+\kappa}}{1+\kappa}\right), \tag{3}$$

as a "depreciation" function of the saving stock, which is convex in the utilization rate (e_t) of savings, analogous to Wen's (1998) model. With this notation and taking the limit $\mathrm{d}t \to 0$, the household budget constraint in Eq. (2) becomes

$$C_t + \dot{S}_t = W_t N_t + \left[e_t R_t^d - \delta(e_t)\right] S_t + \Pi_t. \tag{4}$$

Then we can formulate the constrained optimization problem of the representative household in a continuous-time model as

$$\max \left\{ \int_0^{+\infty} \mathrm{e}^{-\rho t} \left[log\,(C_t) - \psi \frac{N_t^{1+\xi}}{1+\xi} \right] \right\} \tag{5}$$

subject to Eq. (4), where $\rho > 0$ is the discount factor, $\psi > 0$ controls the utility weight on labor supply, and $\xi > 0$ is the inverse Frisch elasticity of labor supply. The FOCs of the household with respect to labor (N_t), consumption (C_t), saving (S_t), and search effort (e_t) are given by

$$\frac{\dot{C}_t}{C_t} = e_t R_t^d - \delta(e_t) - \rho, \tag{6}$$

$$\frac{W_t}{C_t} = \psi N_t^{\xi}, \tag{7}$$

$$R_t^d = \delta'(e_t) = \delta_0 e_t^{\kappa} \tag{8}$$

2.3 Loan search

The loan market consists of a large number of credit lenders (loan officers) and borrowers (firms). More specifically, there are B_t number of loan officers and V_t number of firms. Note that the total deposits in the banking system are given by $\widetilde{S}_t = e_t S_t$, which are divided equally among the loan officers (with measure $B_t = 1$). Each firm must pay a fixed cost ϕ_t to enter the credit market in search of lenders. If a firm is matched with a loan officer, it can produce $y_t = A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha}$ units of output, where n_t is the labor input of the matched firm. The search friction is captured by a matching technology $M(B, V)$, where V denotes the measure of firms entering the credit market after paying the fixed entry cost ϕ. To sharpen the results and for tractability, we also assume a Cobb–Douglas matching technology, $M(B, V) = \gamma B^{1-\varepsilon} V^{\varepsilon}$, with $\varepsilon \in (0, 1)$. Denoting by $\theta_t \equiv \frac{B_t}{V_t}$ a measure of the credit market tightness, the probability that a firm can be matched with a credit supplier is

$$q_t \equiv \frac{M\left(B_t, V_t\right)}{V_t} = M\left(\theta_t, 1\right) = \gamma_t \theta_t^{1-\varepsilon}, \tag{9}$$

and the probability that a loan officer can be matched with a firm is given by

$$u_t \equiv \frac{M\left(B_t, V_t\right)}{B_t} = M\left(1, \frac{1}{\theta_t}\right) = \gamma_t \theta_t^{-\varepsilon}. \tag{10}$$

Note that u_t is also the utilization rate of total bank deposits. That is, the aggregate amount of capital lent out successfully to firms is $u_t \widetilde{S}_t$. Notice that

$$V_t q_t = M\left(B_t, V_t\right) = B_t u_t. \tag{11}$$

Given real wage w_t, if a firm is successfully matched, its operating profit (the matching surplus) is given by $\tilde{\Pi}_t = \max(A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha} - W_t n_t)$. Hence, based on the FOC of n_t, we obtain

$$n_t = \left[A_t \left(\frac{1-\alpha}{W_t}\right)\right]^{\frac{1}{\alpha}} \widetilde{S}_t, \tag{12}$$

$$\tilde{\Pi}_t = \alpha A_t \left[A_t \left(\frac{1-\alpha}{W_t}\right)\right]^{\frac{1-\alpha}{\alpha}} \widetilde{S}_t \equiv \pi_t \widetilde{S}_t. \tag{13}$$

Notice from Eq. (13) that a successfully matched firm's operating profit is proportional to the total bank deposits $\widetilde{S}_t$ (as we are assuming that total bank deposits are divided equally among the loan officers and the measure of loan officers is $B_t = 1$).

For each successful match, the operating profit (surplus) is split between the firm and the loan officer by Nash bargaining, with the firm obtaining $\eta \in [0, 1]$ fraction of the surplus. Denote the competitive interest rate on loans by R_t^l, then it must be true that the lending interest rate equals the expected rate of return to the loan:

$$R_t^l = (1-\eta)\, \pi_t. \tag{14}$$

A firm's *ex ante* expected surplus before conducting credit search is $q_t \eta \pi_t \widetilde{S}_t$. Hence, the free entry (zero profit) condition for the firms is given by

$$\phi_t = q_t \eta \pi_t \widetilde{S}_t. \tag{15}$$

Then Eqs. (9) and (15) together imply

$$q_t = \gamma_t \theta_t^{1-\varepsilon} = \frac{\phi_t}{\eta \pi_t \widetilde{S}_t}. \tag{16}$$

This equation states that the probability of a match for a firm, q_t, decreases with the volume of match surplus. The intuition is as follows. A higher match surplus will induce more firms to enter, hence reducing the probability of a match for each firm with the given number of credit suppliers.

The banking sector is perfectly competitive and thus makes zero profit. The bank pays the depositors the deposit rate R_t^d and earns the rate of return R_t^l (the lending rate) with probability u_t. Therefore, the zero profit condition for the banking sector is given by

$$R_t^d = u_t R_t^l. \tag{17}$$

This equation captures the interest rate spread. Finally, the aggregate net profit income distributed to the household is given by $\Pi_t = (-R_t^d + u_t R_t^l)\widetilde{S}_t + (-\phi + q_t \eta \pi_t) V_t = 0$. Note that although for simplicity we have assumed that the measures of depositors and loan officers are all unity ($H_t = B_t = 1$), the number of firms V_t in the credit market is still a variable.

3 General equilibrium analysis

A general equilibrium is defined as a collection of prices $\{W_t, R_t^d, R_t^l\}$ and quantities $\{C_t, S_t, N_t, V_t, \pi_t, n_t, \tilde{S}_t, K_t, e_t, u_t, q_t\}$ such that (i) given prices and aggregate profit income Π_t, the allocation $\{C_t, S_t, N_t\}$ solves the household's utility maximization problem defined in (5); (ii) the surplus π_t and labor input n_t for a successfully matched firm are defined by (13) and (12); (iii) given the probability u_t of being matched with a bank loan officer, the equilibrium number of firms V_t is determined by the free entry condition (15); (iv) given the bank's probability of being matched with a firm (u_t), the bank earns zero expected profit as characterized by (17); (v) in the credit markets, the probabilities q_t and u_t are determined by (9) and (10); and (vi) both labor markets and goods markets satisfy the standard market-clearing conditions.

3.1 Aggregate production function

Proposition 1 *In general equilibrium, the aggregate production function can be represented by*

$$Y_t = A_t \left(e_t u_t S_t\right)^{\alpha} N_t^{1-\alpha}. \tag{18}$$

Proof See "Appendix." □

To complete the characterization of the aggregate system, we can show that the aggregate surplus of successful matches between loan officers and firms is given by

$$\pi_t \equiv \alpha A_t \left[A_t \left(\frac{1-\alpha}{W_t} \right) \right]^{\frac{1-\alpha}{\alpha}} = \alpha \left(\frac{Y_t}{K_t} \right), \tag{19}$$

which is equal to the marginal product of aggregate capital. The deposit rate is then given by $R_t^d = u_t R_t^l = \alpha(1-\eta)\left(\frac{Y_t}{\widetilde{S}_t}\right)$. The last equality is obtained by combining $K_t = V_t q_t \widetilde{S}_t$ and Eq. (11). Since $B = 1$ and $u_t = \gamma_t \theta_t^{-\varepsilon}$, the aggregate entry costs $V\phi$ satisfy

$$V\phi = \left(\frac{B}{\theta}\right)\phi = \Delta(u) \equiv \Delta_0 \frac{u^{1+\lambda}}{1+\lambda}, \tag{20}$$

where $\Delta(u)$ is convex in u with $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$ and $\lambda \equiv \frac{1}{\varepsilon} - 1 > 0$.

3.2 Local IRS and local indeterminacy

Combining Eqs. (8), (17), (14), and (19) yields

$$e_t = \widetilde{e}\left(\frac{Y_t}{S_t}\right)^{\varepsilon_H}, \tag{21}$$

where $\widetilde{e} \equiv \left(\frac{\alpha(1-\eta)(1-\sigma)}{\delta_0}\right)^{\frac{1}{1+\kappa}}$ and $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. Additionally, the free entry condition (15) implies

$$V\phi = Vq\eta\pi\widetilde{S} = \alpha\eta Y. \tag{22}$$

Combining Eqs. (20) and (22) then yields

$$u_t = \widetilde{u}Y_t^{\varepsilon}, \tag{23}$$

where $\widetilde{u} \equiv \left[\frac{\alpha\eta(1+\lambda)}{\Delta_0}\right]^{\frac{1}{1+\lambda}}$, $\Delta_0 \equiv \frac{\phi(1+\lambda)}{\gamma^{1+\lambda}}$, and $\lambda \equiv \frac{1}{\varepsilon} - 1$.

Proposition 2 *The aggregate production function in Eq. (18) exhibits local IRS (increasing returns to scale) in household capital (S_t) and labor supply (N_t), because it can be rewritten as*

$$Y_t = \bar{Y}A_t^{\tau}S_t^{\alpha_s}N_t^{\alpha_n}, \tag{24}$$

where $\bar{Y} \equiv [(1-\eta)^{\varepsilon_H}(\frac{\eta}{\varepsilon})^{\varepsilon}(\frac{\alpha}{\delta_0})^{\varepsilon_H}(\frac{\alpha\eta}{\Delta_0})^{\varepsilon}]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}}$, $\tau \equiv \frac{1}{1-\alpha(\varepsilon+\varepsilon_H)} > 1$, $\alpha_s \equiv \alpha(1-\varepsilon_H)\tau > \alpha$, *and* $\alpha_n \equiv (1-\alpha)\tau > 1-\alpha$ *with the degree of aggregate returns to scale given by*

$$\alpha_s + \alpha_n > 1. \tag{25}$$

Proof See "Appendix." □

As shown in condition (25), we obtain aggregate IRS in household capital S_t and labor supply N_t despite the lack of Benhabib–Farmer-type production externalities. This is due to the endogenously time-varying utilization rate of aggregate household savings and aggregate bank deposits, as suggested by Eqs. (21) and (23). However, the IRS are *local* because the capital utilization rates, e_t and u_t, are both bounded by the interval [0, 1]. Meanwhile, since $\tau > 1$, we also obtain the amplification effect on productivity shock.

Our model based on credit search appears isomorphic to the IRS models of Benhabib and Farmer (1994) and Wen (1998). Hence, our model may also give rise to local indeterminacy and self-fulfilling business cycles with strong propagation mechanisms as in their models. To see the intuition, consider a proportional increase in aggregate labor and capital supply from the household. In a standard neoclassical model without credit search, such a proportional increase in labor and capital supply would lead, one for one, to an increase in aggregate output one for one. However, in our model, the increase in household savings leads to a larger credit supply in the banking system, which in turn reduces the cost of borrowing and hence induces more firms to enter the credit market. This in turn increases the probability of matching credit resources, raising the effective capital stock used in the production sector by more than the proportional amount, thus resulting in a more than proportionate increase in aggregate output.

In addition to generating the IRS effects, the initial increase in household labor and capital supply can also become self-fulfilling. As the effective capital used in production increases, the returns to labor supply will also increase for every household, reinforcing the initial increase in household labor supply. In addition, as the matching probability of loan officers increases, the bank is able to pay a higher deposit rate. This will increase the returns to saving even for the households that do not increase their savings. Hence, the social IRS originates from a subtle pecuniary externality that reinforces and multiplies itself in a positive feedback loop just like in the model of technological production externalities.

Proposition 3 *The model is locally indeterminate if and only if either of the following conditions hold:*

1. $\alpha \in (0, \frac{1}{2})$, $\xi \in [0, \frac{\alpha}{1-2\alpha})$, $\{\varepsilon, \varepsilon_H\} \in [0, 1]$ *and* $\varepsilon + \varepsilon_H > \widetilde{\varepsilon} \equiv (\frac{1}{\alpha})(\frac{\alpha+\xi}{1+\xi})$.
2. $\alpha \in [\frac{1}{2}, 1)$, $\varepsilon_H \in [0, 1]$ *and* $0 \leq \varepsilon < \frac{1}{\alpha} - 1$.

Proof See "Appendix." □

Some remarks are in order. First, when $\alpha \in (0, \frac{1}{2})$, $\xi \in [0, \frac{\alpha}{1-2\alpha})$, which is line with our calibration in Table 1. Then since $(\frac{1}{\alpha})(\frac{\alpha+\xi}{1+\xi}) \in [1, 2)$, $\eta^* = \frac{\varepsilon}{\varepsilon+\varepsilon_H} < \varepsilon$. The indeterminacy region for this scenario is illustrated in Fig. 3. Second, note that the indeterminacy conditions are unrelated to the bargaining power parameter η. Therefore, indeterminacy always exists under conditions specified in the above proposition

Table 1 Calibration

Parameter	Value	Description
ρ	0.01	Discount factor
A	1	Normalized aggregate productivity
α	0.33	Capital income share
ψ	1.75	Coefficient of labor disutility
ξ	0.2	Inverse Frisch elasticity of labor supply
ε_H	0.82	Matching elasticity in first-stage search
η	0.187	Firm's bargaining power
ϕ	0.086	Vacancy cost of searching for credit.
γ	0.797	Matching efficiency in second-stage search
ε	0.729	Matching elasticity in second-stage search

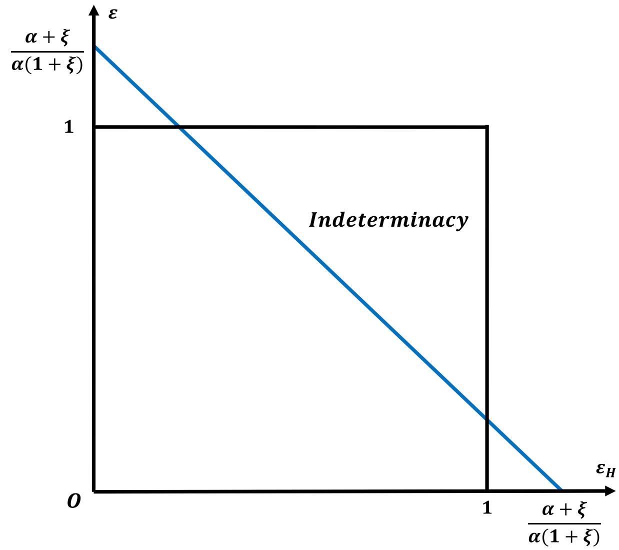

Fig. 3 Indeterminacy region (when $\alpha \in (0, \frac{1}{2})$, $\xi \in [0, \frac{\alpha}{1-2\alpha})$)

regardless of whether we adopt random search with Nash bargaining or competitive search (as shown in the "Appendix"). Third, our model provides a microfoundation to the models of Benhabib and Farmer (1994) and Wen (1998), which rely on exogenously assumed IRS in firms' production technologies. We show, instead, that such IRS technologies can be derived from credit search under CRS technologies and matching functions.

4 Quantitative exercises

4.1 Calibration

The time discounting factor is $\rho = \frac{1}{\beta} - 1 = 0.01$, where $\beta = 0.99$ denotes the discount factor in discrete time models. We set the capital share $\alpha = 0.33$, the coefficient of labor disutility $\psi = 1.75$ and the inverse Frisch elasticity of labor supply $\xi = 0.2$. These values are standard in the existing literature.

Now we must calibrate the values of $(\varepsilon_H, \phi, \eta, \gamma, \varepsilon)$, which are specific to our model. First, as shown in the previous section, $R^d = \frac{\rho(1+\kappa)}{\kappa}$ where $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. We can obtain the average deposit rate R^d from Federal Reserve Economic Data (FRED), which implies $\kappa = 0.23$, and thus $\varepsilon_H = \frac{1}{1+\kappa} = 0.82$. Second, we have proved that $\frac{S}{Y} = \frac{\alpha(1-\eta)}{R^d}$ in the steady state. Consequently, the bargaining power of firms, η, is obtained as $\eta = 1 - (\frac{R^d}{\alpha})(\frac{S}{Y}) = 0.187$.

What remains now is to pin down $(\phi, \gamma, \varepsilon)$. First, we interpret ϕ as the cost of intermediation for financing firm investment, which is related to the size of the financial sector. Philippon (2012) argues that the financial industry is responsible for approximately 8 % of the GDP. Therefore, we set $\frac{\phi}{Y} = 8\,\%$. Note that $\frac{\phi}{Y}$ is related to $(\phi, \gamma, \varepsilon)$.

Second, we set $u = 67\,\%$ according to data on the utilization rate of capital in the manufacturing sector. We also know that u is related to $(\phi, \gamma, \varepsilon)$. Finally, Becchetti et al. (2009) document that approximately 20 % of Italian firms are subject to credit rationing is around 20 %. Assume the rate of credit rationing is 15 % in the USA. Then the proportion of matched firms is $q = \gamma\theta^{1-\varepsilon} = 85\,\%$, where θ is also related to $(\phi, \gamma, \varepsilon)$. Therefore, the three moments $(\frac{\phi}{Y}, u, q)$ jointly imply that $\phi = 0.086$, $\gamma = 0.797$ and $\varepsilon = 0.729$. Our calibration exercise shows that $\varepsilon_H + \varepsilon > (\frac{1}{\alpha})(\frac{\alpha+\xi}{1+\xi})$. Consequently, indeterminacy due to credit search is empirically plausible in our model. The calibration is summarized in Table 1.

4.2 Impulse responses

This subsection investigates the dynamic effect of TFP shocks and matching efficiency shocks (γ_t) on aggregate output, the interest spread, the utilization rate of credit (the opposite of the reserve-to-deposit ratio), and credit rationing. All shocks have AR(1) persistence with the persistence coefficient of 0.9. We discretize our model to facilitate the analysis.

Figure 4 shows the model's impulse responses to a 1 % positive TFP shock. Several striking features of the model are worth mentioning. First, the responses of aggregate output are hump-shaped. Second, there exists a dynamic multiplier-accelerator effect or endogenous propagation mechanism such that not only is the maximum response of output (which far exceeds 1 %) postponed for several periods after the shock is felt, the impact of the shock is also long-lasting with boom–bust cycles or an over-shooting and mean-reverting cyclical pattern. Third, both the reserve-to-deposit ratio and interest spread are countercyclical, consistent with the data.

Similarly, Fig. 5 shows that a 1 % positive shock to the credit matching efficiency can also generate the boom–bust cycles in aggregate output as well as the countercyclical reserve-to-deposit ratio and interest spread. Additionally, Fig. 6 gives the impulse response driven by an *i.i.d.* sunspot shock to labor supply.[6] Despite the lack of any persistence in the sunspot shock, the responses of the economy to such a shock are still highly persistent with boom–bust cycles similar to those under persistent fundamental shocks (except for the initial hump).

To see the intuition, consider an anticipated increase in credit supply from the banking sector. This optimistic expectation would entice firms to increase their search efforts and compete for loans in the credit market, resulting in more banking capital being channeled to firms. Firms can thus increase production by hiring more labor, so households' labor supply would also increase, leading to higher aggregate income and household savings. More household savings would stimulate bank deposits and boost bank's credit supply without creating upward pressure on the loanable funds rate, fulfilling firms' initial optimistic expectations about cheap credit conditions.

[6] Alternatively, the impulse response may also be driven by sunspot shock to consumption demand, which delivers a qualitatively similar result. See Wen (1998) for more details on how to introduce sunspot shocks in indeterminate DSGE models.

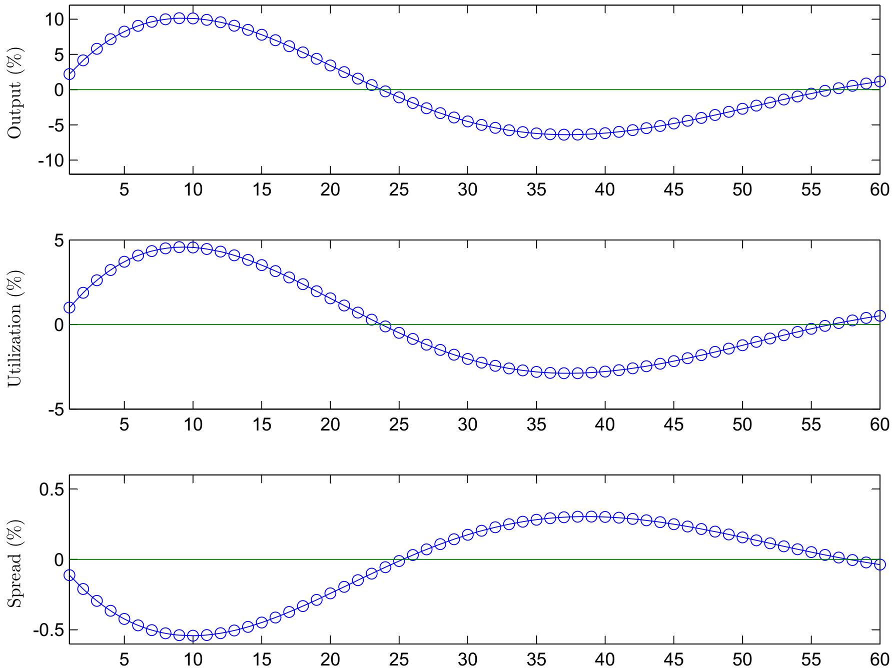

Fig. 4 Impulse responses to positive TFP shock

However, an economic boom, once triggered by a shock (whatever shock it may be), will go through a "natural course" of continuous expansion before returning to the steady state. The positive feedback from firms' production to household income under local IRS means that any increase in firm production would lead to a more than proportionate increase in household savings and bank deposits in the initial periods of the boom, which induces more bank lending and more firms entering in the credit market in search of loans, especially if the shock is expected to persist despite with a damping magnitude.

However, in the absence of permanent productivity (technological) growth, such an economic boom is not sustainable in the long run, because the IRS is only a local property. Once the utilization rate of aggregate credit resources becomes high enough before reaching 100 %, the cost of borrowing will ultimately dominate the rate of return to capital (the marginal product of capital) under diminishing marginal product of capital in the production technology. Hence, the available credit resources in the economy will eventually be exhausted. This implies that after a peak in the boom period, in each subsequent round of feedback between the banking sector and firms, the additional volume of loans unleashed from the banking sector will shrink, ultimately leading to rapid increases in the loanable funds rate. This would eventually choke off credit demand on the firm side because the falling marginal product of capital cannot compensate for the rising costs of credit borrowing. Hence, sooner or later the economy will stop growing and enter a contraction phase.

As the economy continues to contract, the multiplier-accelerator mechanism reverses itself and a persistent period of recession sets in. The recession features a

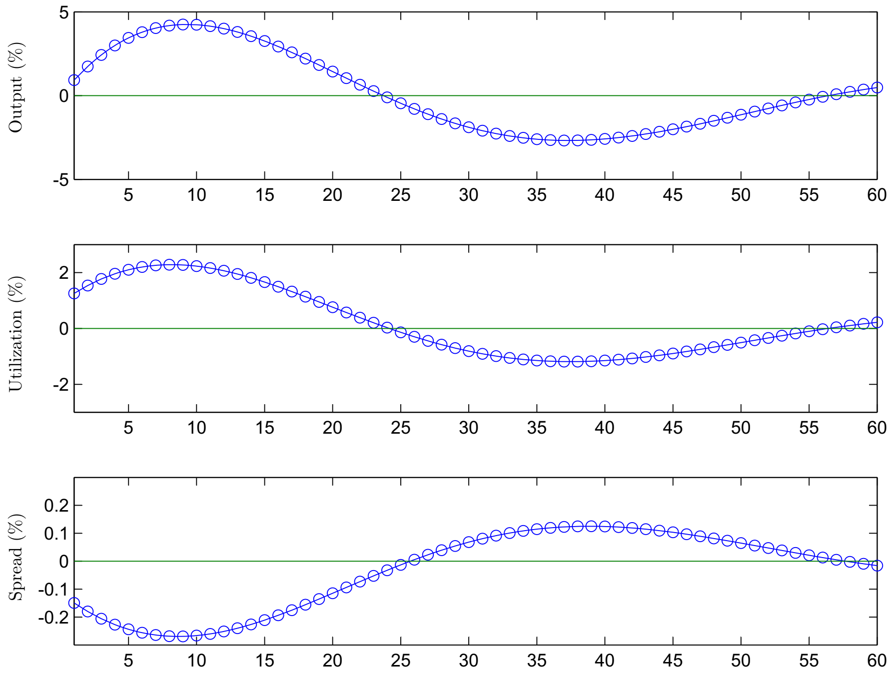

Fig. 5 Impulse responses to a positive shock to credit matching efficiency (γ)

shrinking credit market with firms making little effort to search for credit and excess reserves in the banking sector. However, a prolonged recession cannot be an equilibrium state either, because the economy will eventually reach a point where the marginal product of capital becomes so high (due to a lack of effective investment) and the loanable funds rate becomes so low (due to a lack of credit demand and an increasing excess reserve-to-deposit ratio), which will then cause another round of over-shooting (except with a dampened magnitude) as the economy converges to the steady state again from below.

Hence, our model produces genuine credit cycles broadly consistent with the Austrian theory stating that an economic boom featuring a low interest spread (loanable funds rate below the natural rate) plants the seed for an economic recession, and a recession featuring a high interest spread (loanable funds rate above the natural rate) plants the seed for the next boom. The turning point of the business cycle appears to be determined by the relative magnitudes of the loanable funds rate and the natural rate.

5 Discussions

5.1 Eliminating household search friction

Notice that if $\varepsilon_H = 0$, namely, if there is no household search, then $e_t = 1$ and we obtain $Y_t = A(u_t S_t)^\alpha N_t^{1-\alpha}$, where $u_t = \gamma(\frac{\alpha\eta Y_t}{\phi})^\varepsilon$. Hence, the aggregate production function becomes $Y_t = [\gamma(\frac{\alpha\eta}{\phi})^\varepsilon]^{\alpha_s} A_t^\tau S_t^{\alpha_s} N_t^{\alpha_n}$, where $\alpha_s \equiv \alpha\tau$, $\alpha_n \equiv (1-\alpha)\tau$,

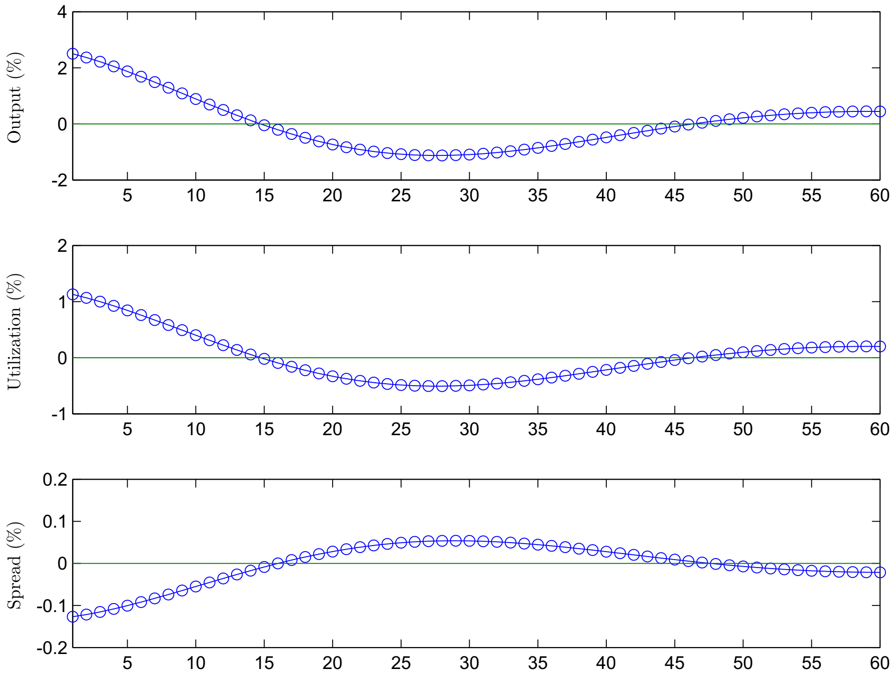

Fig. 6 Impulse responses to sunspot shock

and $\tau \equiv \frac{1}{1-\alpha\varepsilon}$. This production technology still exhibits local IRS because $\alpha_s + \alpha_n = \frac{1}{1-\alpha\varepsilon} > 1$. So the model appears isomorphic to the Benhabib–Farmer model. However, because the IRS are local in our model, whereas they are global in the Benhabib–Farmer model, indeterminacy is not possible in our model with $\varepsilon_H = 0$ *although* we do have endogenous IRS, albeit locally. On the other hand, if we only allow household search but no firm search, i.e., $\varepsilon = 0$ and $\varepsilon_H > 0$, then the model becomes isomorphic to Wen's (1998) model without IRS, which is the Greenwood et al. (1988) model. Hence, adding household search into the model is necessary to generate indeterminacy, analogous to Wen's (1998) finding that variable capacity utilization can significantly reduce the required degree of IRS in the Benhabib–Farmer model for indeterminacy. The well-known problem with the Benhabib–Farmer model is that it requires extremely large IRS to generate indeterminacy, which is empirically implausible. Wen's (1998) model can reduce the required IRS for indeterminacy down to an empirically plausible range. Hence, our model provides a microfoundation for the indeterminacy literature pioneered by Benhabib and Farmer (1994) and Wen (1998) because we show that the Romer-type IRS and Greenwood *et al.*-type capital utilization are not needed to generate essentially identical boom–bust business cycles obtained in Wen (1998).

5.2 Hosios condition and welfare

Since we have used random search to characterize frictions in the credit market, it only makes sense for us to check whether the Hosios (1990) condition holds in our environment. Given (A_t, S_t, N_t), i.e., the technology and the supply of capital and labor, then the Hosios condition is obtained by solving the following constrained optimization problem of the social planner:

$$\max_{e_t, u_t} \{Y_t(e_t, u_t) - \delta(e_t)S_t - \Delta(u_t)\}, \tag{26}$$

where $Y(e, u)$, $\delta(e)$ and $\Delta(u)$ are defined in Eqs. (18), (3) and (20), respectively. Then we reach a modified Hosios condition as follows.

Proposition 4 (modified Hosios condition) *Given (A_t, S_t, N_t), the ratio of output in the decentralized economy to that in the social planner economy is given by*

$$\frac{Y^{\mathrm{DE}}}{Y^{\mathrm{SP}}} = \left[(1-\eta)^{\varepsilon_H}\left(\frac{\eta}{\varepsilon}\right)^{\varepsilon}\right]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}}, \tag{27}$$

and thus

$$\eta^* = \arg\max_{\eta\in[0,1]}\left(\frac{Y^{\mathrm{DE}}}{Y^{\mathrm{SP}}}\right) = \frac{\varepsilon}{\varepsilon+\varepsilon_H}. \tag{28}$$

Several remarks are in order. First, when search frictions exist only between banks and firms, i.e., $\varepsilon_H = 0$, then $Y^{\mathrm{DE}} = Y^{\mathrm{SP}}$ if and only if $\eta = \varepsilon$, which shares resembles the classic Hosios condition. However, in contrast to the classic Hosios condition, $\eta = \varepsilon$ does not maximize Y^{DE}. Second and more interestingly, we contribute to the literature by detecting a modified Hosios condition in the presence of dual search frictions, i.e., $\varepsilon_H > 0$ and $\varepsilon > 0$. Intuitively, the increase in a firm's bargaining power η delivers two competing effects. On the one hand, the increase in η intensifies a firm's search for credit by inducing more firm entry. This in turn increases u, the utilization rate of credit in the second link of the search and matching chain, thus driving up output. On the other hand, a higher η diminishes the profit share of the bank by lowering the loan rate, which translates into a lower deposit rate, and therefore discourages the household from making a deposit. Therefore Eq. (28) strikes a balance between these two competing effects. In particular, η^* increases with ε (the matching elasticity of firms searching for credit) and decreases with ε_H (the matching elasticity of the household searching for financial intermediation). Notice that $\eta^* = \varepsilon$ if and only if $\varepsilon + \varepsilon_H = 1$. As shown in the next subsection, $\varepsilon + \varepsilon_H > 1$ when indeterminacy emerges, and thus $\eta^* < \varepsilon$ in that scenario.

Finally, in deriving the Hosios conditions, we have followed the literature by holding the supply of labor and capital fixed. This restriction is fine when it comes to the standard setup of macrolabor economics *a la* DMP, which typically assumes inelastic labor supply and risk-neutral firms and workers, but usually ignores capital accumulation. However, our paper must address both of these issues since the household in our model is allowed to decide on labor supply as well as capital accumulation. Moreover, the household is risk averse when it comes to consumption. As a result, neither the classic nor the modified Hosios condition can guarantee a constrained optimum in welfare. Instead, we have to take into account the effect of η on both the consumption and leisure decisions of the household over the lifetime horizon. Note that the household's welfare in the steady state is

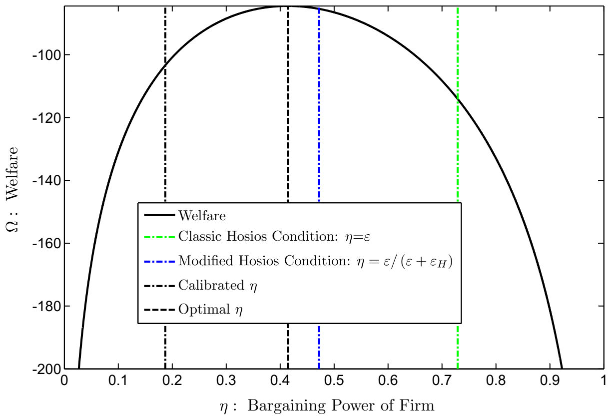

Fig. 7 Hosios conditions and welfare: a numerical example (see Table 1 for parameterization)

$$\Omega = \frac{1}{\rho}\left\{\log\left[\left(\frac{C}{Y}\right)Y\right] - \psi\frac{N^{1+\xi}}{1+\xi}\right\},$$

where $\frac{C}{Y} = (1 - \frac{\alpha}{1+\kappa}) - (\frac{\kappa}{1+\kappa})\alpha\eta$, $N = (\frac{1-\alpha}{\psi}\frac{1}{C/Y})^{\frac{1}{1+\xi}}$, and $Y = [A(\gamma(\frac{\alpha\eta}{\phi})^{\varepsilon})^{\alpha}(\frac{S}{Y})^{\alpha} N^{1-\alpha}]^{\frac{1}{1-\alpha(1+\varepsilon)}}$ denote, respectively, the steady state of the ratio of consumption to output, labor supply and output, all of which are obtained from the simplified dynamical system in the proof of Proposition 3. Figure 7 indicates that given risk aversion, endogenous capital accumulation, and elastic labor supply, neither the classic Hosios condition (i.e., $\eta = \varepsilon$) nor the modified Hosios condition (i.e., $\eta = \frac{\varepsilon}{\varepsilon+\varepsilon_H}$) maximizes the household's welfare in the steady state.

5.3 Competitive search

For simplicity, we have adopted random search in the baseline model. Alternatively, we can use competitive search *a la* Moen (1997). More specifically, each loan officer can set his/her own terms of trade, R^l, in sub-market θ such that

$$\max\left\{u(\theta)R^l(\theta)\widetilde{S}\right\} \tag{29}$$

subject to

$$q(\theta)\left[\pi - R^l(\theta)\right]\widetilde{S} = \phi \quad \text{for all } \theta, \tag{30}$$

where $u(\theta) = \frac{M(B(\theta),V(\theta))}{B(\theta)}$, $q(\theta) = \frac{M(B(\theta),V(\theta))}{V(\theta)}$. If $M(B,V) = \gamma B^{1-\varepsilon}V^{\varepsilon}$, then it is easy to see that the equilibrium loan rate is determined by $R^l = (1-\varepsilon)\pi$, which is qualitatively similar to the loan rate under Nash bargaining in Eq. (14). Additionally, we can check that the indeterminacy condition characterized in Proposition 3 still holds. Therefore, the sunspot condition is unrelated to the bargaining protocol.

6 Conclusion

The critical role that credit supply and financial intermediation play in generating and amplifying the business cycle has long been observed by economists at least since the Austrian school, as manifested historically in the countercyclical excess reserve-to-deposit ratio, the countercyclical interest rate spread between the loan rate and the deposit rate, and the countercyclical proportion of firms subject to credit rationing. However, a simple correlation between credit expansion/contraction and economic boom/bust does not imply causality. This paper provides a framework to rationalize the Austrian theory and the observed credit cycles. Our framework is based on a simple idea: In an industrial economy where labor is divided and decision making is segregated between consumption demand and output supply and between household saving and firm investment, savers (lenders) with "idle" credit resources must search for matching investors (borrowers) who have projects that can utilize the available saving/credit resources and make them productive. But search and matching are costly due to informational frictions and various transaction costs and commitment technologies. They also require efforts and coordination between borrowers and lenders. Hence, in equilibrium, credit resources in the economy are not always fully utilized, creating an important margin for elastic credit supply—excess reserves and an endogenous utilization rate of available credit resources. So economic booms and busts are closely associated with credit expansions and contractions and changes in interest spread. Meanwhile, the under-utilization of credit resources coexists with the prevalence of credit rationing to firms. Our highly stylized model nonetheless demonstrates the fundamental nature of credit-driven economic boom–bust cycles. Finally, we show that such a margin of elastic credit supply turns out to be critical not only for understanding the countercyclical excess reserve-to-deposit ratio and interest rate spread, but also for providing a microfoundation for the powerful amplification and propagation mechanisms underlying the endogenous business cycle literature studied by Benhabib and Farmer (1994) and Wen (1998) based on the assumption of IRS in production technologies.

Acknowledgments We benefit from comments by the anonymous referee, Costas Azariadis, Silvio Contessi, Bill Dupor, Francois Geerolf (discussant), Rody Manuelli, Benjamin Pugsley, B. Ravikumar, Yi-Chan Tsai (discussant), José-Víctor R íos-Rull, Harald Uhlig, Randy Wright, Steve Williamson, as well as participants of 2015 ASSA meeting at Boston, the macroseminar at Federal Reserve Bank of St. Louis, the NBER conference on Multiple Equilibria and Financial Crises at Federal Reserve Bank of San Francisco, Tsinghua Workshop in Macroeconomics 2015, and The Fourth HKUST International Workshop on Macroeconomics. The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. The usual disclaim applies.

Appendix: Proofs

Proof of Proposition 1 Since each match between the household and the banking sector utilizes $\widetilde{S}_t = e_t S_t$ units of household capital (savings), and each match between the banking sector and the production sector (firms) utilizes $K_t = u_t \widetilde{S}_t$ units of bank capital (deposits), given the total initial available credit resources S_t in the economy, the fraction of aggregate credit resources ultimately utilized in production is hence given by

$$K_t = u_t e_t S_t. \tag{31}$$

As each matched firm employs n_t units of labor, the labor market equilibrium then requires

$$N_t = V_t q_t n_t = V_t q_t \left[A_t \left(\frac{1-\alpha}{W_t} \right) \right]^{\frac{1}{\alpha}} \widetilde{S}_t. \tag{32}$$

Finally, it is easy to show that the total production output of all firms is given by

$$Y_t = V_t q_t y_t = V_t q_t A_t \widetilde{S}_t^{\alpha} n_t^{1-\alpha} = A_t \left(V_t q_t \widetilde{S}_t \right)^{\alpha} N_t^{1-\alpha} = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{33}$$

where K_t is determined by Eq. (31). Since $K_t = e_t u_t S_t$, the aggregate production function in Eq. (33) can also be written as in Eq. (18). □

Proof of Proposition 2 Substituting Eqs. (21) and (23) into Eq. (18) yields Eq. (24). □

Proof of Proposition 3 Equations (12) and (32) together imply

$$W_t = (1-\alpha) \left(\frac{Y_t}{N_t} \right). \tag{34}$$

Substituting Eq. (34) into Eqs. (4), (6), and (7) yields

$$\dot{S}_t = (1 - \alpha\eta) Y_t - \delta(e_t) S_t - C_t, \tag{35}$$

$$\frac{\dot{C}_t}{C_t} = (1-\eta)\alpha \left(\frac{Y_t}{S_t} \right) - \delta(e_t) - \rho, \tag{36}$$

$$\psi N_t^{\xi} = (1-\alpha) \left(\frac{Y_t}{N_t} \right) \left(\frac{1}{C_t} \right). \tag{37}$$

Consequently, we can reduce the dynamic system $\{C_t, S_t, N_t, W_t, R_t^d, R_t^l, \pi_t, K_t, e_t, u_t, q_t, \theta_t, Y_t, V_t\}$ to a simplified one with fewer variables in $\{C_t, S_t, e_t, u_t, Y_t, N_t\}$ with Eqs. (18), (21), (23), (35), (36), and (37), where $\delta(e)$ is defined in Eq. (3). The FOCs indicate $\delta'(e_t) = (1+\kappa)\left(\frac{\delta(e_t)}{e_t}\right) = R_t^d$. Thus,

$$\delta(e_t) = \frac{R_t^d e_t}{1+\kappa} = \varepsilon_H \alpha (1-\eta) \left(\frac{Y_t}{S_t} \right).$$

Log-linearizing the above simplified transition dynamics yields

$$\begin{aligned}
\dot{c}_t &= (1-\varepsilon_H)\left(\frac{Y}{S}\right)(1+\widehat{y}_t-\widehat{s}_t)-\rho \\
\dot{s}_t &= [(1-\alpha\eta)-\varepsilon_H\alpha(1-\eta)]\left(\frac{Y}{S}\right)(1+\widehat{y}_t-\widehat{s}_t)-\left(\frac{C/Y}{S/Y}\right)(1+\widehat{c}_t-\widehat{s}_t) \\
\widehat{y}_t &= \alpha(\widehat{e}_t+\widehat{u}_t+\widehat{s}_t)+(1-\alpha)\widehat{n}_t \\
\widehat{e}_t &= \varepsilon_H(-\widehat{s}_t) \\
\widehat{u}_t &= \varepsilon\widehat{y}_t \\
(1+\xi)\widehat{n}_t &= (1-\alpha)(\widehat{y}_t-\widehat{c}_t)\,.
\end{aligned}$$

Consequently, we obtain the simplified dynamic system on (s_t, c_t) as

$$\begin{bmatrix}\dot{s}_t\\ \dot{c}_t\end{bmatrix}=J\cdot\begin{bmatrix}\widehat{s}_t\\ \widehat{c}_t\end{bmatrix},$$

where

$$J\equiv\delta\cdot\begin{bmatrix}\left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)\lambda_s & \left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)(\lambda_c-1)\\ \kappa(\lambda_s-1) & \kappa\lambda_c\end{bmatrix}.$$

$\kappa\equiv\frac{1}{\varepsilon_H}-1$, $\alpha_s\equiv\frac{\alpha(1-\varepsilon_H)}{1-\alpha(\varepsilon+\varepsilon_H)}$, $\alpha_n\equiv\frac{1-\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}$, $\lambda_s\equiv\frac{\alpha_s(1+\xi)}{1+\xi-\alpha_n}$, and $\lambda_c\equiv\frac{-\alpha_n}{1+\xi-\alpha_n}$.

Note that the local dynamics around the steady state is then determined by the eigenvalues of J. If both eigenvalues of J are negative, then the model is indeterminate. As a result, the model can experience endogenous fluctuations driven by sunspots. The eigenvalues of J, x_1 and x_2 satisfy

$$\begin{aligned}
x_1+x_2 &= \text{Trace}(J)=\delta\left[\left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{1}{1-\eta}\right)\lambda_s+\kappa\lambda_c\right],\\
x_1x_2 &= \text{Det}(J)=\delta^2\left(\frac{1+\kappa}{\alpha}-1\right)\left(\frac{\kappa}{1-\eta}\right)(\lambda_s-\lambda_c-1)\,.
\end{aligned}$$

Therefore, indeterminacy emerges if and only if $\text{Trace}(J)<0$ and $\text{Det}(J)>0$. We can prove that $\text{Trace}(J)<0$ and $\text{Det}(J)>0$ if and only if the following four conditions hold, in addition to the restriction that $\varepsilon,\varepsilon_H\in[0,1]$:

$$\varepsilon+\varepsilon_H<\frac{1}{\alpha}, \tag{38}$$

$$\varepsilon+\varepsilon_H>\left(\frac{1}{\alpha}\right)\left(\frac{\alpha+\xi}{1+\xi}\right), \tag{39}$$

$$\varepsilon_H<1-\frac{(1-\eta)(1-\alpha)\kappa}{(1+\kappa-\alpha)(1+\xi)}, \tag{40}$$

$$\varepsilon<\frac{1}{\alpha}-1. \tag{41}$$

First, since $\varepsilon_H \in [0, 1]$, comparing Conditions (38) and (41) suggests that the former is never binding. Second, note that $\kappa \equiv \frac{1}{\varepsilon_H} - 1$. Thus, Condition (40) can be rewritten as

$$\varepsilon_H < \left[\frac{1+\xi-(1-\eta)(1-\alpha)}{1+\xi}\right]\left(\frac{1}{\alpha}\right).$$

Since $\xi \geq 0$, we have $[\frac{1+\xi-(1-\eta)(1-\alpha)}{1+\xi}](\frac{1}{\alpha}) > [1-(1-\eta)(1-\alpha)](\frac{1}{\alpha}) > 1$, and therefore, we know that Condition (40) is not binding. Finally, if $\alpha \in [\frac{1}{2}, 1)$, then we know that $\frac{1}{\alpha} - 1 \in (0, 1]$, and we must have $0 \leq \varepsilon < \frac{1}{\alpha} - 1$. Besides, we know that $\widetilde{\varepsilon} \equiv (\frac{1}{\alpha})(1 - \frac{1-\alpha}{1+\xi}) > 2$ when $\alpha \in [\frac{1}{2}, 1)$. Therefore, Condition (39) always holds in this case. In contrast, when $\alpha \in (0, \frac{1}{2})$, we have $\frac{1}{\alpha} - 1 > 1 > \varepsilon$, and thus, Condition (41) always holds. Meanwhile, since $\varepsilon + \varepsilon_H \leq 2$, to guarantee that Condition (39) can be satisfied, we must have $\widetilde{\varepsilon} \equiv (\frac{1}{\alpha})(1 - \frac{1-\alpha}{1+\xi}) < 2$, i.e., $\xi \in [0, \frac{\alpha}{1-2\alpha})$. □

Proof of Proposition 4 The FOCs are given by

$$\delta^0 e_t^{\kappa} = \frac{\alpha Y_t}{e_t S_t}, \tag{42}$$

$$\Delta^0 u_t^{\lambda} = \frac{\alpha Y_t}{u_t}. \tag{43}$$

Substituting Eqs. (42) and (43) into Eq. (18) yields

$$Y^{\text{SP}} = \widetilde{Y}^{\text{SP}} A_t^{\tau} S_t^{\alpha_s} N_t^{\alpha_n}, \tag{44}$$

where $\widetilde{Y}^{SP} = [(\frac{\alpha}{\delta^0})^{\varepsilon_H}(\frac{\alpha\eta}{\Delta^0})^{\varepsilon}]^{\frac{\alpha}{1-\alpha(\varepsilon+\varepsilon_H)}}$. Dividing Eq. (24) by Eq. (44) yields Eq. (27). Then the FOC of Eq. (27) yields η^*. □

References

Acemoglu, D.: A microfoundation for social increasing returns in human capital accumulation. Q. J. Econ. **111**(3), 779–804 (1996)

Azariadis, C., Kaas, L., Wen, Y.: Self-fulfilling credit cycles. Working Paper, Federal Reserve Bank of St. Louis (2014)

Bai, Y., Ríos-Rull, J.V., Storesletten, K.: Demand shocks as productivity shocks. Working Paper, Federal Reserve Board of Minneapolis (2012)

Becchetti, L., Garcia, M., Trovato, G.: Credit rationing and credit view: empirical evidence from loan data. Working Paper, Tor Vergata University, CEIS (2009)

Benhabib, J., Farmer, R.E.: Indeterminacy and increasing returns. J. Econ. Theory **63**(1), 19–41 (1994)

Benhabib, J., Dong, F., Wang, P.: Adverse selection and self-fulfilling business cycles. NBER Working Paper (2014a)

Benhabib, J., Miao, J., Wang, P.: Chaotic banking crises and banking regulations. NBER Working Paper (2014b)

Benhabib, J., Wang, P.: Financial constraints, endogenous markups, and self-fulfilling equilibria. J. Monet. Econ. **60**(7), 789–805 (2013)

Campello, M., Graham, J.R., Harvey, C.R.: The real effects of financial constraints: evidence from a financial crisis. J. Financ. Econ. **97**(3), 470–487 (2010)

Cui, W., Radde, S.: Search-based endogenous illiquidity and the macroeconomy. Working Paper, UCL (2014)

Den Haan, W.J., Ramey, G., Watson, J.: Liquidity flows and fragility of business enterprises. J. Monet. Econ. **50**(6), 1215–1241 (2003)
Duffie, D., Gârleanu, N., Pedersen, L.H.: Over-the-counter markets. Econometrica **73**(6), 1815–1847 (2005)
Gertler, M., Kiyotaki, N.: Banking, liquidity and bank runs in an infinite-horizon economy. NBER Working Paper (2013)
Greenwood, J., Hercowitz, Z., Huffman, G.W.: Investment, capacity utilization, and the real business cycle. Am. Econ. Rev. **78**(2), 402–417 (1988)
Hosios, A.J.: On the efficiency of matching and related models of search and unemployment. Rev. Econ. Stud. **57**(2), 279–298 (1990)
King, R.G., Rebelo, S.T.: Resuscitating real business cycles. Handb. Macroecon. **1**, 927–1007 (1999)
Lagos, R., Rocheteau, G.: Liquidity in asset markets with search frictions. Econometrica **77**(7), 403–426 (2009)
Liu, Z., Wang, P.: Credit constraints and self-fulfilling business cycles. Am. Econ. J. Macroecon. **6**(1), 32–69 (2014)
Miao, J., Wang, P.: Bubbles and credit constraint. Working Paper, Boston University and Hong Kong University of Science and Technology (2012)
Moen, E.R.: Competitive search equilibrium. J. Polit. Econ. **105**(2), 385–411 (1997)
Petrosky-Nadeau, N., Wasmer, E.: The cyclical volatility of labor markets under frictional financial markets. Am. Econ. J. Macroecon. **5**(1), 193–221 (2013)
Philippon, T.: Has the US finance industry become less efficient? On the theory and measurement of financial intermediation (No. w18077). National Bureau of Economic Research (2012)
Pintus, P.A., Wen, Y.: Leveraged borrowing and boom-bust cycles. Rev. Econ. Dyn. **16**(4), 617–633 (2013)
Stiglitz, J.E., Weiss, A.: Credit rationing in markets with imperfect information. Am. Econ. Rev. **71**(3), 393–410 (1981)
Wasmer, E., Weil, P.: The macroeconomics of labor and credit market imperfections. Am. Econ. Rev. **94**(4), 944–963 (2004)
Wen, Y.: Capacity utilization under increasing returns to scale. J. Econ. Theory **81**(1), 7–36 (1998)